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Jump Phenomena in Y-Shaped Intake Ducts

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Nomenclature

A	= duct area
f, g	= functions
M	= Mach number
m	= mass flow rate
P, p	= total and static pressure, respectively
R	= gas constant
S	= sections of duct
T	= total temperature
γ	= ratio of specific heats
μ	= mass flow factor

Subscripts

C	= compressor face
E	= entry station
l, u	= lower, upper branch of solution
M	= mixing station
1, 2, 3	= duct section 1, 2, 3

Introduction

THE air intake of a fighter aircraft must meet the engine mass flow demand over a range of aircraft speeds and attitudes[1] with high total pressure recovery and low distortion. Y-shaped ducts are a popular choice for air intakes in single-engined fighter aircraft. The intakes are normally side-mounted and the two limbs of the duct merge inside the fuselage into one and feed the engine. Y-shaped ducts are normally expected to operate in a steady, symmetric manner. In this case, the engine mass flow demand is met by the two limbs by inducing equal mass flows, each being half of what the engine requires.

Received Feb. 13, 1995; revision received Sept. 8, 1995; accepted for publication Sept. 10, 1995.

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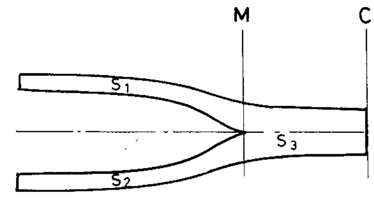

Fig. 1 Y-shaped duct, sections S_1, S_2, and S_3, and stations E, M, and C.

Steady, asymmetric operation where the two limbs induce unequal mass flows, though not immediately obvious, can never be ruled out. Martin and Holzhauser[2] reported such an operation as early as 1950. The flow in this case, even if smooth in the individual ducts, can be expected to be highly distorted on mixing. The available duct length within the fuselage is very likely to be insufficient to smooth out the distortion before the flow reaches the engine face. This Note proposes a simple flow model that explains the phenomenon that causes transition from symmetric to asymmetric operation.

Flow Model

The duct is made up of three sections (Fig. 1). The first two sections, S_1 and S_2, are the two symmetric limbs up to the station where they merge. Section S_3 is the region where the two flows mix and does not play a major role in the analysis that follows. Stations E, M, and C (Fig. 1) are the entry, the merging plane, and the compressor face, respectively. Each duct has a performance characteristic in the form of total pressure ratio across it against mass flow through it. Sections S_1 and S_2 can have an asymmetric operation if, for a given total entry pressure P_E, the mixing plane static pressure p_M is the same for both sections.

For S_1 (or S_2) one can estimate p_M/P_E for any given mass flow rate m_1 (or m_2) as follows: The total pressure ratio is known from the duct characteristic while the total temperature remains unchanged throughout the duct:

$$P_M/P_E = f(m), \qquad T_M = T_E \tag{1}$$

For a given P_E and T_E, and for A_M known from the duct geometry, one can use Eq. (1) to calculate the mass flow factor $\mu = m\sqrt{T_M}/(A_M P_M)$. Mach number M_M is solved for using the compressible flow relation:

$$\sqrt{\gamma/R}M_M = \mu\{1 + [(\gamma - 1)/2]M_M^2\}^{(\gamma+1)/2(\gamma-1)} \tag{2}$$

Thus, we can get

$$\begin{aligned} p_M/P_E &= (p_M/P_M)(P_M/P_E) \\ &= \{1 + [(\gamma - 1)/2]M_M^2\}^{-\gamma/(\gamma-1)}(P_M/P_E) = g(m) \end{aligned} \tag{3}$$

Consider an engine mass flow rate demand of m. This can be met by m_1 and m_2 through S_1 and S_2 as follows:

$$m = m_1 + m_2, \qquad p_{M_1}/P_E = g_1(m_1), \qquad p_{M_2}/P_E = g_2(m_2) \tag{4}$$

The flows m_1 and m_2 will have to satisfy the compatibility condition that $p_{M_1} = p_{M_2}$, which means the static pressure at the mixing station is the same for both limbs. This leads to the following condition, which is a nonlinear equation in one variable m_1 and a parameter m:

$$g_1(m_1) - g_2(m - m_1) = 0 \tag{5}$$

It is assumed in the previous discussion that the characteristic of S_1 and S_2 may, in general, be different. If g_1 and g_2 are identical functions, then $m_1 = m/2$ is a solution of Eq. (5), which represents symmetric operation. For a given m, Eq. (5) can, in general, yield more than one solution for m_1 and these may be investigated using any continuation algorithm.[3] The average pressure recovery of the duct is calculated as

$$\frac{P_M}{P_E} = \frac{(P_{M_1}/P_E)m_1 + (P_{M_2}/P_E)m_2}{m_1 + m_2} \tag{6}$$

Bifurcation Analysis

Consider the case where f_1 and f_2 are identical functions that represent parabolically decreasing pressure recovery with increasing mass flow. In addition, if $A_{M_1} = A_{M_2}$, then g_1 and g_2 will be identical functions. For such a pressure loss characteristic, the Y-shaped duct has only symmetric operation throughout the mass flow range. This is true for any characteristic with monotonically decreasing pressure recovery with an increase in mass flow. Consider another case where f_1 and f_2 are identical functions as shown in Fig. 2. This variation is close to the one indicated by Seddon and Goldsmith[4] and can be taken to be a reasonable approximation to the actual pressure recovery characteristic of a duct. Calculations are made for $T_E = 288$ K, $P_E = 101{,}325$ Pa, and $A_M = 0.3$ m^2.

Figures 3 and 4 show variations of m_1 and P_M/P_E with m for steady-state operation of this intake. Multiple operating points for a range of engine demands m can be seen, with only one of them ($m_1 = m/2$) corresponding to symmetric operation. Stable operating branches are shown in full lines and unstable branches are shown in dotted lines. The different bifurcation points in Figs. 3 and 4 with their types are A and D = subcritical pitchfork bifurcation and B_u, B_l, C_u, and C_l = saddle-node bifurcation.

Consider a duct in stable, symmetric operation at a large value of m. This symmetric solution is often called the primary solution. As m decreases, one encounters the pitchfork bifurcation at A where the primary symmetric solution loses stability. With a slight perturbation, the system then jumps to the stable asymmetric operating point at H_u or H_l, depending on the direction of the perturbation. That is, when the perturbation is such as to cause m_1 to jump to H_u, it will also result in m_2 jumping to H_l so that Eq. (4) is satisfied. The onset of asymmetric flow occurs at point A which, by comparing the P_M/P_E curve in Fig. 4 with that in Fig. 2, can be seen to correspond to the peak of the pressure recovery characteristic in Fig. 2.

On further decrease of m, stable asymmetric operation persists until the system reaches points C_l, C_u, which are saddle-node bifurcation points (also called limit points or turning points). At these points, the system jumps back to the stable symmetric solution at I. Similarly, for increasing values of m starting from a stable symmetric solution at a low mass flow rate, one encounters the pitchfork bifurcation at D where the primary symmetric solution loses stability and the system jumps to stable asymmetric operation at J_l, J_u. On further increase in m, asymmetric operation is maintained until the saddle-node points at B_l, B_u are reached, when the system jumps back to the symmetric solution at K. Thus, for decreasing values of m in Figs. 3 and 4, duct operation is along K–A–H–J–C–I and for increasing values of m, it is along I–D–J–H–B–K.

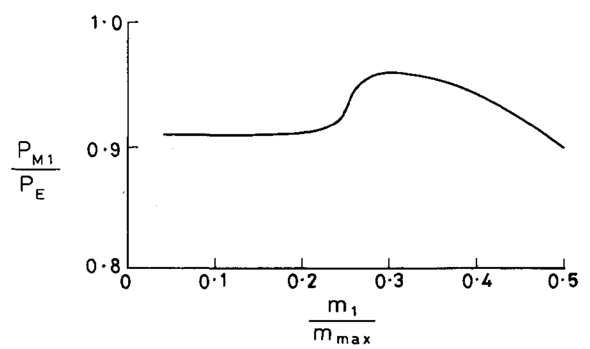

Fig. 2 Pressure recovery characteristic f_1 (or f_2).

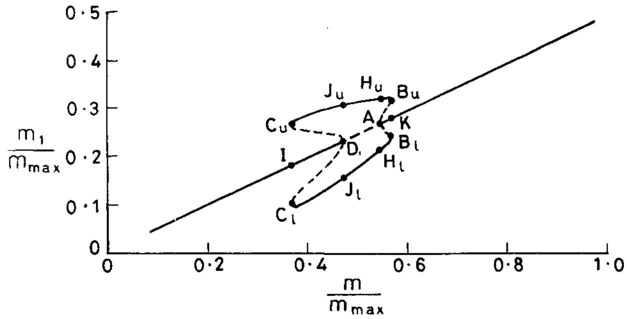

Fig. 3 Steady-state operation, variation of m_1.

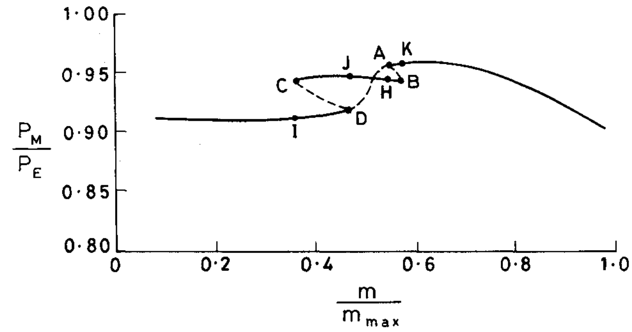

Fig. 4 Steady-state operation, variation of average duct pressure recovery.

Discussion

At supersonic speeds, when mass flow is decreased, the normal shock at the intake entry gets expelled and moves ahead of the intake. At low enough mass flows, it can step down to the fuselage from the intake splitter plate. The shock-boundary-layer interaction and ingestion of the separated boundary layer can result in a drop in pressure recovery, (e.g., as in Fig. 2) which, from the discussion in the previous section, may be used as a criterion for onset of asymmetric operation.

Conclusions

A simple flow model is proposed for a Y-shaped duct that predicts the boundary for stable symmetric operation marked by a pitchfork bifurcation (point A). Study has indicated that this bifurcation takes place as long as the pressure recovery characteristic has a maximum. One physical phenomenon that can give rise to such behavior is discussed. This provides intake designers with a useful criterion for ensuring symmetric operation of intakes in supersonic flight.

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